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Flow stress, strain rate, strain relationships in superplastic deformation

W. R. KERR
Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio 45433, USA

The superplastic deformation of materials has commonly been characterized by the strain rate sensitivity, *m*, determined from two-dimensional, flow stress against strain rate plots. For material in which flow stress varies with strain or because of microstructural changes due to time at a high temperature, superplastic deformation must be characterized with respect to a three dimensional plot in which strain or time is the third axis. Techniques for generating the necessary plots are indicated and a simple experimental method for determining the suitability of a given material to a particular forming operation is described.

1. Introduction

Superplasticity, the capacity of a metal to achieve extensive tensile elongation, usually at elevated temperature, is frequently defined in terms of strain rate sensitivity, m, defined as

$$m = \frac{\mathrm{d}\ln \sigma_\mathrm{f}}{\mathrm{d}\ln \dot{\epsilon}}, \tag{1}$$

where σ_f is the flow stress and $\dot{\epsilon}$ the strain rate. Superplasticity is generally indicated by values of $m \geqslant 0.5$. For the relationship between flow stress and strain rate Backofen, *et al.* [1] used the equation

$$\sigma_\mathrm{f} = K(\dot{\epsilon})^\mathrm{m}. \tag{2}$$

Flow stress against strain rate data were obtained from step strain rate tests [1] in which a tensile specimen was subjected to varying pulling speeds and the stress at which the load became level at each speed was plotted against the strain rate for that speed in the form $\ln \sigma_\mathrm{f}$ versus $\ln \dot{\epsilon}$. A quantitative value of m was then determined from the slope of the curve thus generated.

Paton [2] has written a further equation which fits the curves generated by the step strain rate technique

$$\dot{\epsilon} = K_1(\sigma - \sigma_0) + K_2\sigma^N, \tag{3}$$

where K_1, K_0, K_2 and N are constants. A computer is then used to fit load and cross-head speed data to this equation and to determine the slope of the curve.

These equations are an adequate description of the variation of flow stress with strain rate "under conditions which do not favour work hardening of the material" [2] or where the material is not subject to microstructural changes during deformation. Lee and Backofen [3] found, that for the titanium alloys they considered, the true stress–true strain curves were flat over the range of 30 to 40% strain involved in a full determination of m against $\dot{\epsilon}$.

However, where material is subject to strain hardening or to microstructural changes during deformation, the above equations are inadequate and variation in flow stress due to these other variables must be considered. This suggests an equation of the form

$$\sigma = K_3\dot{\epsilon}^m + K_4\epsilon^n, \tag{4}$$

where K_3, K_4, m and n are constants. Microstructural changes are not explicitly dealt with in this equation. However, such changes may result from exposure time at the elevated temperatures where superplastic deformation is commonly achieved. Since, at a given strain rate, the strain is a unique function of time, microstructural changes due to time at high temperature are accounted for implicitly in the strain term, ϵ. The equation might therefore also be written

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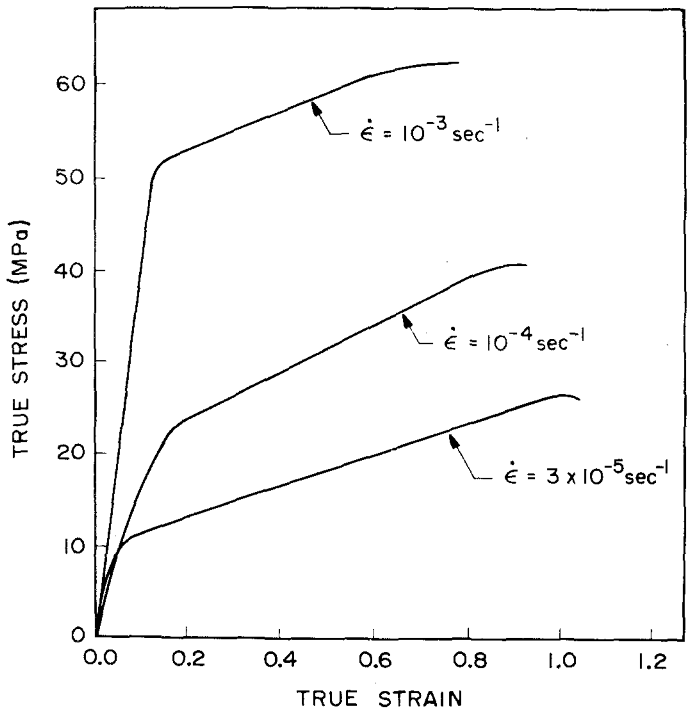

Figure 1 True stress against true strain curve from constant strain rate tests. Determined from Ti–6 wt% Al–4 wt% V sheet, grain size = 18 μm, at 870° C. (From [8]).

$$\sigma = K_1 \dot{\epsilon}^m + K_2 t_\epsilon^{n'} \tag{5}$$

where strain is implicit in the time, t.

These latter equations indicate that for materials subject to change during deformation, the relationship between flow stress and strain rate is not determined by the two dimensional curve indicated by Equation 3, but by a three dimensional surface with the time or strain as the third axis. Further, m must be defined with respect to constant strain and microstructure as indicated by Suery and Baudelet [4]. Since microstructural changes with time are uniquely related to strain for a given strain rate

$$m = \left(\frac{\mathrm{d}\ln \sigma_f}{\mathrm{d}\ln \dot{\epsilon}}\right)_\epsilon . \tag{6}$$

2. Results

The superplastic behaviour of titanium alloys, especially of Ti–6 wt%Al–4 wt%V, is being utilized in the fabrication of aircraft structures [5, 6]. Data generated on this alloy indicate that the material is subject to significant, microstructural changes during deformation [7–9]. Fig. 1 shows that, at constant strain rate, flow stress increases appreciably with strain (and time). These data, from [8], were obtained from a Ti–6 wt% Al–4 wt%V sheet being considered for superplastic deformation; the grain size was large (approximately 18 μm) and the superplasticity of the material somewhat limited, since necking occurred at true strains less than 1.0. The data were generated at 870° C.

Fig. 2 shows a three dimensional plot of the data of Fig. 1. Constant stress contours have been superimposed on this plot. This surface may be considered as characteristic of the superplastic behaviour of the material at the test temperature if it is assumed that within the narrow limits of the tests, flow stress is a unique function of strain and strain rate. It may be generated, as in this example, by constant strain rate tests in which each strain has a flow stress uniquely associated with it. The surface might also be generated by constant stress tests in which each strain is uniquely associated with a strain rate. In laboratory tests for superplasticity, strain rate is commonly the independent variable. In actual forming operations, pressure, hence stress, is the controlled variable. In this case, the way in which strain rate and strain vary with applied stress, i.e. the way in which the material behaviour "moves" on the plotted surface, will depend upon the rate at which stress is applied, $\dot{\sigma}$. This can be seen by considering an instantaneous application of stress, in which case strain rate will increase parallel to the constant strain contours; instantaneous removal of stress will result in movement to zero strain rate, also along a constant

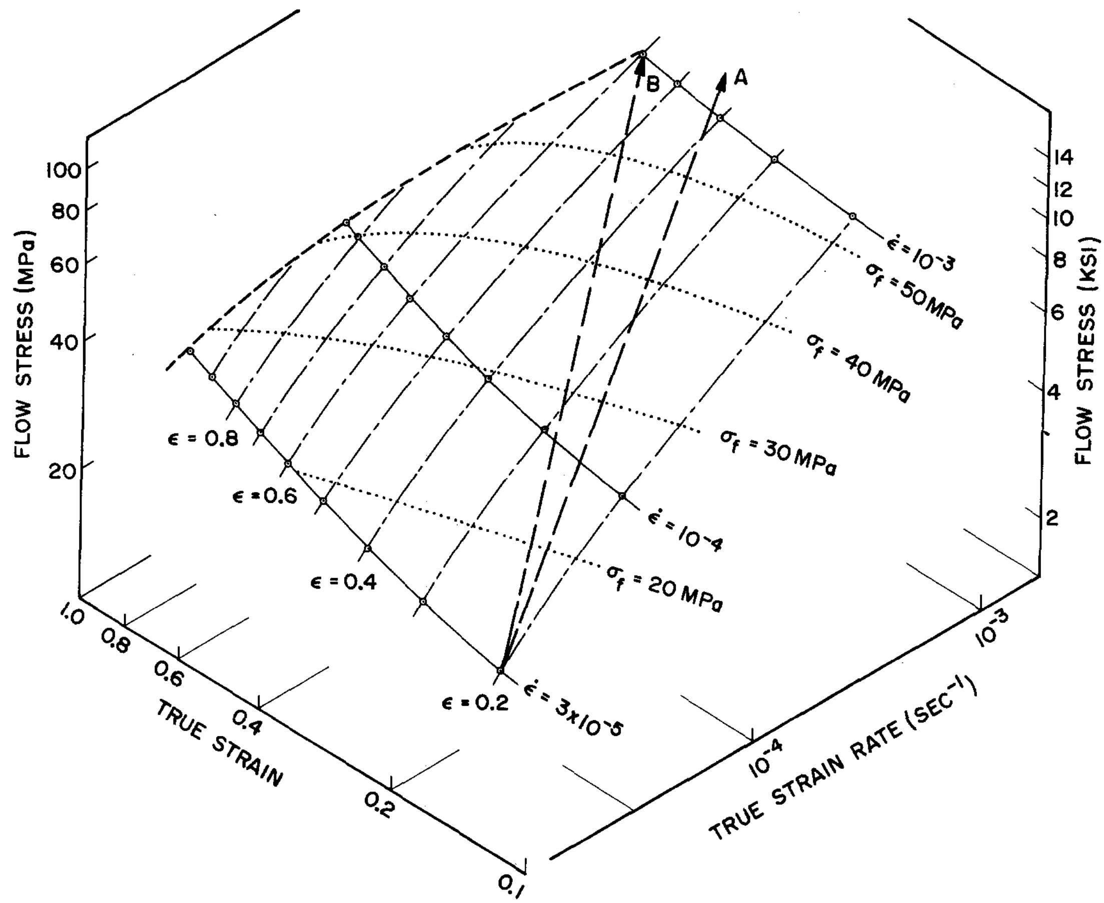

Figure 2. Three dimensional plot of σ_f, $\dot{\epsilon}$ and ϵ data from Fig. 1. Vector A represents a stress rate which would result in failure of a formed part; Vector **B**, the optimum stress rate.

strain contour; maintenance of constant stress will result in movement to higher strain and lower strain rate along a constant stress contour. It should be noted that actual movement on the surface must always be toward higher strain. This surface of superplastic behaviour is limited by the strain, at any strain rate, at which necking occurs, and by the strain rate at which strain rate sensitivity falls below some critical value and necking occurs. The diagram may therefore be used to indicate the limits of superplastic deformation for this material.

It is apparent that Equations 2 and 3 would apply only if the constant strain rate and constant stress contours were both parallel to the strain axis. This would be indicated by horizontal lines after the "knees" in Fig. 1, i.e. constant stress for each strain rate in the region of plastic strain. The surface apparently fits an equation of the form of Equation 4 but no attempt was made to evaluate the terms of this equation. From the curvature of the constant strain contours it can be seen that m is a function of strain rate. The linear relationship between stress and strain in Fig. 1 indicates that the exponent n is unity for this material but the change in slope with strain rate indicates that K_2 is a function of strain rate.

Fig. 3 shows the projection of the constant strain contours of the superplastic deformation surface on the $\ln \sigma_f$–$\ln \dot{\epsilon}$ plane. A hypothetical step strain rate curve is superimposed on this plot. (See also Fig. 14 of [7]). Since each increase in flow stress involves an increment due to increased strain as well as an increment due to increased strain rate, the results obtained from such tests are, in part, characteristic of the test and the strain imposed at each strain rate. Where strain at each step is small and the material is not subject to significant changes during the test, m values determined by this technique may not apply at the large strains often encountered in actual forming operations. When the material being deformed is

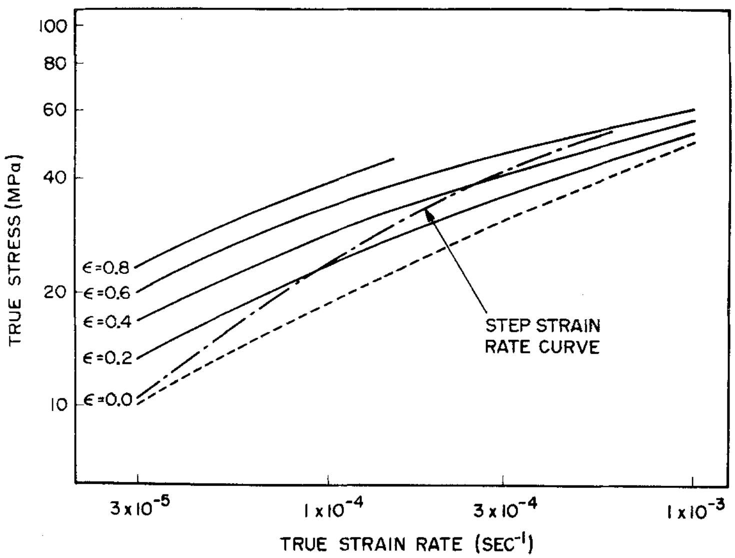

Figure 3 Projection of the superplastic deformation surface on the σ_f against $\dot{\epsilon}$ plane. Hypothetical data from a step strain rate test is superimposed.

subject to changes due to strain (or time), m values determined without regard to strain,

$$m = \frac{\mathrm{d}\ln\sigma_\mathrm{f}}{\mathrm{d}\ln\dot{\epsilon}}$$

would tend to be inflated. Values determined from the slope of the constant strain curves,

$$m = \left(\frac{\mathrm{d}\ln\sigma_\mathrm{f}}{\mathrm{d}\ln\dot{\epsilon}}\right)_\epsilon ,$$

would be more characteristic of the material, independent of test techniques.

3. Discussion

Generation of the superplastic deformation surface for a particular material would be useful in the study of superplastic behaviour and the interrelation of the effects of strain and microstructural changes during deformation. However, a generation of the surface is not necessary to determine whether a particular material is suitable for use in a given forming operation, i.e. as a quality control test.

Since the variation in strain and strain rate depends only upon the stress rate, it is necessary only to know the critical stress rate which a material can withstand in achieving the necessary strain. This is illustrated by the two vectors shown in Fig. 2. It is assumed that fabrication of a part requires a true strain, $\epsilon = 0.6$, and that the strain rate, $\dot{\epsilon} = 10^{-3}$ represents the onset of necking, then a stress rate along the vector **A** would result in the onset of necking before the required strain was reached, a stress rate along the vector **B** would be considered optimum in terms of forming the part in minimum time, hence maximizing facility efficiency and minimizing microstructural changes due to time at high temperature. Slightly lower stress rates would offer a margin for error.

A simple method for determining stress rate is available. Jovane [10] has written an equation for the stress in a membrane blown into a hemispherical die by gas pressure

$$\sigma = \frac{R}{4t_0}\frac{(1+H)^2}{H}P, \tag{7}$$

where H is h/R, h is the distance the membrane is blown into the die, R is the radius of the die, t_0 is the initial thickness of the sheet and P is the pressure. Mackay *et al.* [8] have shown that for a conical die with an appropriate cone angle, the factor $[(1+H^2)^2/H]$ is approximately constant. Then

$$\sigma = \frac{R}{t_0}KP \tag{8}$$

and stress rate may be determined from pressurization rate. If a die with several conical cavities of differing radii is used, a number of stress rate curves may be determined from a single test. Proper choice of cone radii would permit determination of the critical stress rate for the application being considered.

4. Conclusions

(a) For materials subject to microstructural changes and/or strain hardening during superplastic deformation the relationship between flow stress and strain rate is determined not by a two-dimensional curve but on a three-dimensional surface with strain or time as the third axis.

(b) In forming operations, variation in flow stress, strain rate and strain, i.e. movement on the three dimensional surface, is determined by the rate of application of stress.

(c) The suitability of a particular material for a particular superplastic forming operation depends upon the critical stress rate, the maximum stress rate which can be applied to reach the required strain without the onset on necking.

(d) Critical stress rate can be determined easily from the cone test.

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